Arithmetic

Numbers

Definitions

Natural: 0, 1, 2, 5, 9, 103, …; notation **N**
Integers: …, -67, -4, 0, 6, 89, … (natural, positive and negative); notation **Z**
Rational: 3/4, -5/8, 59/34, 4.29, -5.782, … (also known as "fractions"), notation **Q**
Decimal: 3.56, -9.604, … (whole part plus decimal/fractional part)
Conversion between rational and decimal example: 3/4 = .75; 4.57 = 457/100
Irrational (transcendental), p = 3.14159; notation **I**
Note: they can NOT be converted to rational numbers! (without losing precision)
Real: Natural + Integers + Rational + Irrational; notation **R**
Complex: 3 + i7, -5 + i2, -i5, …, where i = v-1 initially called "imaginary" number; notation **C**; C = R + pure "imaginary" numbers (containing i)

Properties

Prime numbers = a natural number that can not be divided by any other number except for 1 and itself.

Factoring: decomposition of any Natural number into prime numbers. Example: 12 = 2 * 2 * 3, where 2 and 3 are prime numbers.

Operations with numbers: addition, subtraction, multiplication and division. It works "more of less different" depending on the type of numbers involved.

Least common multiplier and **larger common denominator**.

Famous Constants

Pythagoras' Constant (from the name of a famous Greek mathematician)

The diagonal of a unit square has length v2 = 1.14142135624... A theory, proposed by the Pythagorean school of philosophy, maintained that all geometric magnitudes could be expressed by rational numbers. The sides of a square were expected to be commensurable with its diagonals, in the sense that certain integer multiples of one would be equivalent to integer multiples of the other. This theory was shattered by the discovery that v2 is irrational.

The Golden Mean ?

We start with a problem in aesthetics. Consider the following line segment:

____

What is the most "pleasing" division of this line segment into two parts? Some people might say at the halfway point:

__________________x_________________

Others might say at the one-quarter or three-quarters point. The "correct answer" is, however, none of these, and is found in Western art from the ancient Greeks onward (art theorists speak of it as the principle of "dynamic symmetry"):

_______________________x____________

Here, if the left-hand portion is of length u = 1, then the right-hand portion is of length v = 0.6180339887... A line segment partitioned as such is said to be divided in Golden or Divine section. What is the justification for endowing this particular division with such elevated status? The thinking is that the length u, as drawn, is to the whole length u+v, as the length v is to u. In symbols,

u/(u + v) = v/u
Letting ? = u/v, we solve for ? by observing that
$?^{2} - ? - 1 = 0$
So, $?_{1}$ = (1 + v5)/2 = 1,6180339887…, and $?_{2}$ = (1 - v5)/2 = 0.6180339887…

The Natural Logarithmic Base

e = 2.7182818284... discovered by the Scottish mathematician John Napier.

e = $\lim_{n->8}$ S (1/n!)

Archimedes' Constant

p = 3.141592653… discovered by the famous Greek mathematician when dividing the perimeter of a circle by its diagonal.

Numbering Systems

Definitions

Base or *radix*: 10, 2, 8, 16, etc.
General *representation*: $378 = 3 * 10^2 + 7 * 10^1 + 8 * 10^0$ also known as "*positional*" notation
Conversions: using the "*modulo algorithm*"

Number Base Systems - Example

decimal	binary	ternary	octal	hexadecimal
0	0	0	0	0
1	1	1	1	1
2	10	2	2	2
3	11	10	3	3
4	100	11	4	4
5	101	12	5	5
6	110	20	6	6
7	111	21	7	7
8	1000	22	10	8
9	1001	100	11	9
10	1010	101	12	A
11	1011	102	13	B
12	1100	110	14	C
13	1101	111	15	D
14	1110	112	16	E
15	1111	120	17	F
16	10000	121	20	10
17	10001	122	21	11
18	10010	200	22	12
19	10011	201	23	13
20	10100	202	24	14

Average (Mean)

Definitions

Arithmetic: 1/n * ? (x_i), i = 1, …, n, and x_i can be any value; example: 1/3 (5 + 6 + 7) = 6
Geometric: n^{th} root of ? x_i, i = 1, …, n, and x_i can be any value; example: 3^{rd} root of (1 * 5 * 25) = 5
Harmonic: n / ? $(1/x_i)$, i = 1, …, n, and x_i can be any value; example: 2/(1/2 + 1/3) = 12/5 = 2.4

Weighted: 1/n * ? (w_i * x_i), i = 1, …, n, and x_i can be any value; example: 1/4 (2*3 + 2*1 + 4*2 + 0*15) = 4

Progressions

Definitions

Arithmetic: a, a + 2r, a + 3r, a + 4r, …, a + (n-1)r, where a and r can be any value

Geometric: a, a * r, a * r^2, a * r^3, a * r^4, …, a * $r^{(n-1)}$, where a and r can be any value

Harmonic: 1/a, 1/(a + r), 1/(a + 2r), 1/(a + 3r), …, 1/(a (n-1)r), where a and r can be any value

Series

Definitions

Fibonacci (from the name of a famous Italian mathematician) series: F_0 = 0, F_1 = 1, F_2 = F_0 + F_1, …, F_n = F_{n-1} + F_{n-2}

One of very interesting properties of this series is that $\lim_{n->8}$ F_n/F_{n-1} = ? (the *Golden Mean*)!

Geometry

Point, Line, Curves

Definitions

1, 2, 3, …, n dimensions

Properties

Continuity
Concavity vs. *convexity*

Parallelogram

Perimeter: P = 2 * (length + width)
Area: A = base * height

Rectangle

Perimeter: P = 2 * (length + width)
Area: A = length * width

Square

Perimeter: P = 4 * side
Area: A = $side^2$

Trapezoid

Perimeter: P = 2 * (length + width)
Area: A = 1/2 * height * (base1 + base2)

Triangle

Perimeter: P = side1 + side2 + side3
Area: A = 1/2 * side * height

Circle

Perimeter: $P = 2 * p * radius$
Area: $A = p * radius^2$

Ellipse

Perimeter: $P = 2 * p * v1/2(radius1^2 + radius2^2)$
Area: $A = p * radius1 * radius2$

Cylinder

Volume: $V = p * radius^2 * height$
Surface: $S = 2 * p\ radius * height$

Sphere

Volume: $V = 4/3 * p * radius^3$
Surface: $S = 4 * p * radius^2$

Cone

Volume: $V = 1/3\ p\ radius^2 * height$
Surface: $S = p\ radius * v(radius^2 + hieght^2)$

Pyramid

Volume: $V = 1/3 * S * height$
Surface: S = depends on its shape; if a square, then $S = side^2$

Cube

Volume: $V = side * side * side$, or $side^3$
Surface: $S = 6 * side^2$

Algebra

Proportions

Definitions

$a/b = c/d$

Properties

$(a + b) / b = (c + d) / d$
$(a - b) / b = (c - d) / d$
$(a - b) / (a + b) = (c - d) / (c + d)$

Powers and Roots

Definitions

$x^n = x * x * x * \ldots x$ (n times); x is called the *base* and n the *exponent*; the entire expression is called a *power*.

n^{th} root of x equals a number that raised to power of n will produce x.

Properties

$x^a * x^b = x^{(a + b)}$
$x^a * y^a = (x * y)^a$
$(x^a)^b = x^{(ab)}$
$x^{(a/b)} = b^{th}$ root of $(x^a) = (b^{th}\ v(x)\ )^a$
$x^{(-a)} = 1 / x^a$
$x^{(a - b)} = x^a / x^b$

Logarithms

Definitions

$y = \log_b (x)$ if and only if $x = b^y$

Properties

$\log_b (1) = 0$
$\log_b (b) = 1$
$\log_b (x*y) = \log_b (x) + \log_b (y)$
$\log_b (x/y) = \log_b (x)-\log_b (y)$
$\log_b (x^n)=n \log_b (x)$
$\log_b (x) = \log_b (c) * \log_c (x) = \log_c (x) / \log_c (b)$
$\ln = \log_e$ (natural logarithm), where e = 2.7182818284...
$\lg = \log_{10}$

Vectors

Definition

$V = (a_1, a_2, a_3, \ldots, a_n)$, where n is a Natural number

Operations

Multiplication by a scalar: $k * V = (k * a_1, k * a_2, k * a_3, \ldots, k * a_n)$, where n and k are Natural numbers
Addition/subtraction of a scalar: $k + V = (k + a_1, k + a_2, k + a_3, \ldots, k + a_n)$, where n and k are Natural numbers
Addition/subtraction of two vectors: $V + W = (a_1 + b_1, a_1 + b_2, a_1 + b_3, \ldots, a_1 + b_n)$, where n is a Natural number, and $V = (a_1, a_2, a_3, \ldots, a_n)$ and $W = (b_1, b_2, b_3, \ldots, b_n)$

Matrices & Determinants

Definition

Example: A 3 x 3 matrix looks like

$$A_{3,3} = \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

or, in general, $A_{n,m} = \{a_{i,j} \mid i = 1, \ldots, n \text{ and } j = 1, \ldots, m; n, m \text{ e } N\}$

Operations

Addition of two matrices: (note both matrices MUST be of same dimension, e.g., 3 x 3 in the following example,

$A_{3,3} + B_{3,3} = C_{3,3}$, as follows:

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix} \quad + \quad \begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix} \quad = \quad \begin{matrix} (a_{11} + b_{11}) & a_{12} + b_{12}) \; a_{13} + b_{13}) \\ (a_{21} + b_{21}) & a_{22} + b_{22}) a_{23} + b_{23}) \\ (a_{31} + b_{31}) & a_{32} + b_{32}) a_{33} + b_{33}) \end{matrix}$$

Multiplication by a scalar:

$k * A_{3,3} = B_{3,3}$ as follows:

$$B_{3,3} = \begin{matrix} k * a_{11} & k * a_{12} & k * a_{13} \\ k * a_{21} & k * a_{22} & k * a_{23} \\ k * a_{31} & k * a_{32} & k * a_{33} \end{matrix}$$

Multiplication of two matrices: $A_{n,m} * B_{m,p} = C_{n,p}$ where $C_{n,p} = \{c_{i,j} = ?_{k=1,m}\, a_{ik} * b_{kj} \mid i = 1, \ldots, n$ and $j = 1, \ldots, p; n, p$ e $N\}$

Combinatorics

Factorial

Definitions

n! = 1 * 2 * 3 * … * n
Recursively, n! = n * (n-1)
n! can be approximated by *Stirling*'s formula (famous English mathematician), n! = e^{-n} * n^n * 2 vp * n, where e and p are two of the famous constants explained above.

Arrangements or Permutations

Definitions

P(n,m) = n! / (n - m)! = n * (n-1) * … * (n - m + 1)

Combinations

Definitions

C(n,m) = n! / [m! * (n - m)!] = P(n,m) / m!

Properties

C(n,m) + C(n,m+1) = Cn+1,m+1
C(n,m) = C(n,n-m)

Binomial formula (Newton's - famous English mathematician*)*: $(1 + x)^n$ = Cn,0 * x^0 + Cn,1 * x^1 + Cn,2 * x^2 + … + Cn,n * x^n

Pascal's triangle - famous French mathematician: Cn,0 + Cn,1 + … + Cn,n = 2^n

Randomization

Definitions

A *random number* is a number chosen arbitrarily. It can be pulled from already developed tables or using a *randomizing function* or *algorithm*.

Sorting Algorithms

<u>Definitions</u>

Arranging a string of values in a given (ascending or descending) order.

<u>Algorithms</u>:

Quick Sort (*Hoare*'s algorithm - famous English mathematician)

"Bubble" Sort

This area to be covered (in more detail) during the class, as possible.

Data Structures

This area to be covered during the class, as possible.

Types

Definitions

- Stack
- Queue
- Linked List
  - Simple
  - Double
- Trees
  - Binary
  - Balanced

Properties

- Stack
- Queue
- Linked List
  - Simple
  - Double
- Trees
  - Binary
  - Balanced

Operations

- Search
- Insert
- Delete
- Update
- Traverse

File Organization & Access

This area to be covered during the class as possible.

Declaration

Definitions

Layout
Buffering

Media

Definitions

Tape
Diskette
Disc

Properties

Tape
Diskette
Disc

Organization

Definitions

Sequential
Indexed-Sequential
Direct/Random

Properties

Sequential
Indexed-Sequential
Direct/Random

Access

<u>Definitions</u>

Sequential
Indexed (multi)
Direct

<u>Properties</u>

Sequential
Indexed (multi)
Direct

Maintenance

<u>Definitions</u>

<u>Issues</u>

Operations

<u>Definitions</u>

Open
Read
Write
Update
Append
Close

<u>Properties</u>

Open
Read
Write
Update
Append
Close